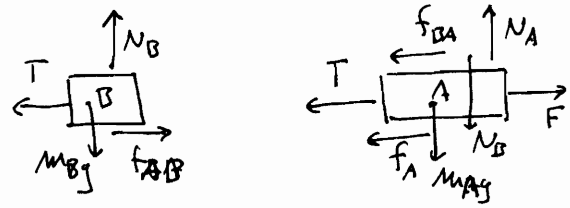
PHY105 Final – Solutions 2005

① A) FBD



$|f_{BA}| = |f_{AB}|$

For const vel $a = 0$

Block B $\quad T - f_{AB} = 0 \Rightarrow T = f_{AB} = \mu_2 N_B = \mu_2 m_B g$

Block A $\quad F - f_A - T - f_{BA} = 0 \Rightarrow F = f_A + 2|f_{AB}|$

$$= \mu_1 N_A + 2\mu_2 m_B g$$

$$N_A = (N_B + m_A g) = (m_A + m_B) g \Rightarrow$$

$$F = \mu_1 (m_A + m_B) g + 2\mu_2 m_B g = 76\,N$$

B) If $T_{max} = 100\,N$

$$\begin{cases} T - f_{AB} = m_B a \\ F - T - f_A - f_{BA} = m_A a \end{cases} \qquad \vec{a}_A = -\vec{a}_B$$

$$\Rightarrow \quad F = T\left(1 + \frac{m_A}{m_B}\right) + f_A + f_{AB}\left(1 - \frac{m_A}{m_B}\right) = 539\,N$$

c) $$f = \frac{W_{friction}}{W_{Force}} = \frac{\text{Ans A}}{\text{Ans B}} = \frac{76}{539} = 14\%$$

or $$f = \frac{f_A d + f_{AB}(2d)}{F d}$$

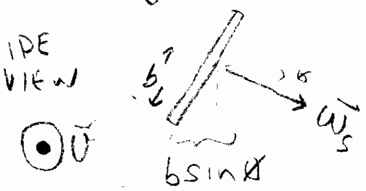
② From K+K pg 123

Conservation of momentum. If V_r is the recoil velocity of the block in the x direction, then the speed of the marble with respect to the table is $V_0 \cos\theta - V_r$

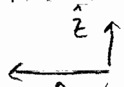
$$\Rightarrow \quad M V_r = m(V_0 \cos\theta - V_r) \quad (\text{ie } P_{gun} = P_{marble})$$

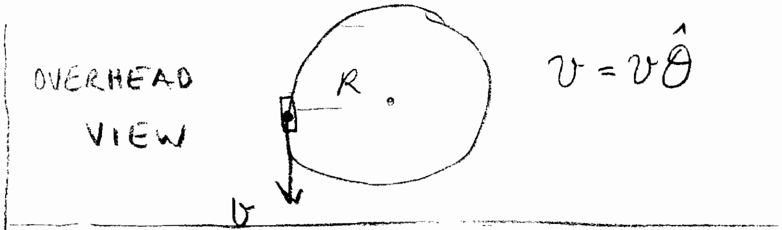
$$\Rightarrow V_r = \frac{m V_0 \cos\theta}{m + M}$$

3. Rolling coin, radius $b \ll R$, speed v.



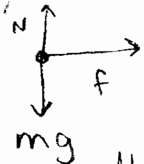
POLAR COORDINATES AT THIS POSITION:





Coin rolls without slipping so it has a spin component to its angular frequency $\omega_s = v/b$, directed as sketched: $\vec{\omega}_s = -\omega_s \cos\phi\,\hat{r} - \omega_s \sin\phi\,\hat{z}$.

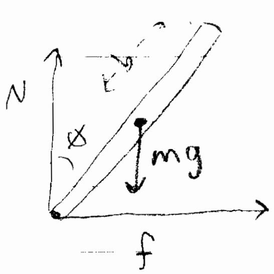
The CM travels in circle of radius $r_{CM} = R - b\sin\phi \simeq R$ since $b \ll R$, with angular frequency $\vec{\Omega} = v/R\,\hat{z}$. Consider Newton's law for the CM:



We require friction $f = mv^2/R$ to provide centripetal acceleration. $\sum F_z = 0$ so $N = mg$.

Next consider the torque equation. We must choose an origin about which to calculate $\vec{\tau}$ and $\vec{L}$. It must be EITHER the CM or the origin of an inertial frame: We cannot choose the point of contact to the table! (Note that the fictitious "centrifugal" force arising from working in the noninertial frame of the CM acts at the CM, so its associated $\vec{\tau}$ there is zero.)

Here we choose the CM (but append a solution with a different origin.)



$$\sum \vec{\tau} = f b\cos\phi\,\hat{\theta} - N b\sin\phi\,\hat{\theta} = mb\left(\frac{v^2\cos\phi}{R} - g\sin\phi\right)\hat{\theta}$$

$\vec{L} = L_s\cos\phi(-\hat{r}) + L_s\sin\phi(-\hat{z}) + L_{side}\,\hat{z}$, so that

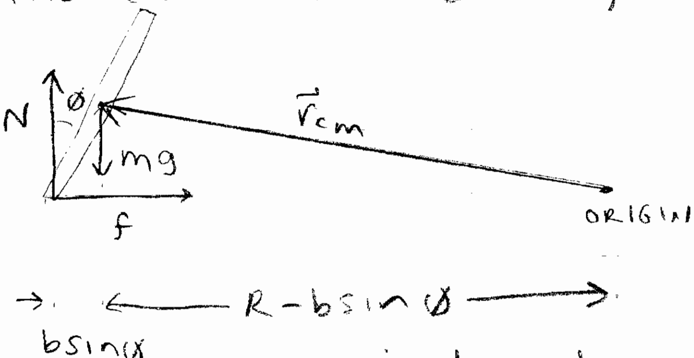
$$\frac{d\vec{L}}{dt} = L_s\cos\phi\left(-\frac{d\hat{r}}{dt}\right) = -\Omega L_s\cos\phi\,\hat{\theta},$$

where L_s is the angular momentum due to the spin: $\vec{L}_s = I_0\vec{\omega}_s$, so that $L_s = \left(\frac{1}{2}mb^2\right)(v/b) = mb(v/2)$. Here $L_{side} = I'\Omega$ arises because the coin also spins about a vertical axis through its CM as it negotiates the circle. (This is easiest to visualize for $R \to 0$:)

So $\dfrac{d\vec{L}}{dt} = -\Omega L_s\cos\phi\,\hat{\theta} = -mb\left(\dfrac{v^2}{2R}\right)\cos\phi\,\hat{\theta}$. We take $\sum\vec{\tau} = d\vec{L}/dt$ & find ϕ.

$$mb\left(\frac{v^2}{R}\cos\phi - g\sin\phi\right) = -mb\left(\frac{v^2}{2R}\cos\phi\right) \Rightarrow \boxed{\frac{3}{2}\frac{v^2}{Rg} = \tan\phi}$$

Now let us consider the case where the inertial frame has its origin at the center of the circle, on the table. (Another good choice is at the center, but at the height of the CM above the table, $z = b\cos\phi$.)



$$\sum\vec{\tau} = \vec{r}_{CM}\times mg(-\hat{z}) + RN(-\hat{\theta})$$
$$= mg(R-b\sin\phi)\hat{\theta} - Rmg\hat{\theta}$$
$$= -mbg\sin\phi\,\hat{\theta}$$

Now we calculate $\vec{L}'$, which again has terms due to spin, and to I', and also a term due to the angular momentum of the CM with respect to this origin:

$$\vec{L}'_{CM} = m\vec{r}_{cm}\times\vec{v} = m\left[(R-b\sin\phi)\hat{r} + b\cos\phi\,\hat{z}\right]v\times\hat{\theta}$$
$$= mv(R-b\sin\phi)\hat{z} - mbv\cos\phi\,\hat{r}$$

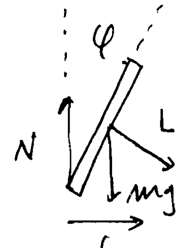
So the total angular momentum is of the form:

$$\vec{L}' = \vec{L}_s + \vec{L}'_{side} + \vec{L}'_{CM} = (-L_s\cos\phi - mbv\cos\phi)\hat{r} + L_z\hat{z}$$

so that
$$\frac{d\vec{L}'}{dt} = \left(-mb\left(\frac{v}{2}\right)\cos\phi - mbv\cos\phi\right)\Omega\,\hat{\theta} = -\frac{3}{2}\frac{mbv^2\cos\phi}{R}$$

and we see from $\sum\tau' = \frac{dL'}{dt}$ that $\tan\phi = \frac{3}{2}\frac{v^2}{Rg}$ again.

(3) For full soln see PS#6. Here is another soln



Forces in radial direction $f = \frac{mv^2}{R}$ $(b \ll R)$

Forces in normal direction $N = mg$

Torque about cm: $\tau = fb\cos\varphi - Nb\sin\varphi$ (out of page) $\hat{\theta}$

L has three components. The only one that changes is $L\cos\varphi\,\hat{r}$ so lets look at that $L_r = -I\omega\cos\varphi\,\hat{r}$

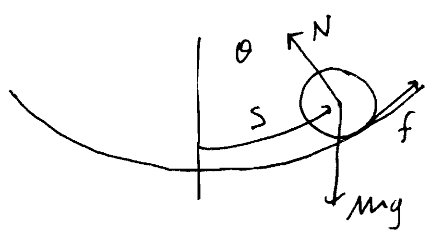
$$\vec{\tau} = \frac{d\vec{L}}{dt} = -I\omega\Omega\cos\varphi\,\hat{\theta} \qquad \text{where } \omega = v/b \;;\; \Omega = v/R$$

Combining $-I\omega\Omega\cos\varphi = fb\cos\varphi - Nb\sin\varphi$

or $Nb\tan\varphi = fb + \frac{Iv^2}{bR} \Rightarrow \tan\varphi = \frac{\cancel{m}v^2\cancel{b}}{\cancel{m}g\cancel{b}R} + \frac{1}{2}\frac{\cancel{b^2}mv^2}{\cancel{b^2}RN}$

$$\Rightarrow \varphi = \tan^{-1}\left(\frac{3}{2}\frac{v^2}{gR}\right)$$

(4) Rolling ball



Forces in $\hat{\theta}$ direction:

$-mg\sin\theta + f$ = net

Force in $\hat{r}$ direction $mg\cos\theta = N$

Torque $I\alpha = fb$ $\quad \alpha = -\ddot{s}/b$

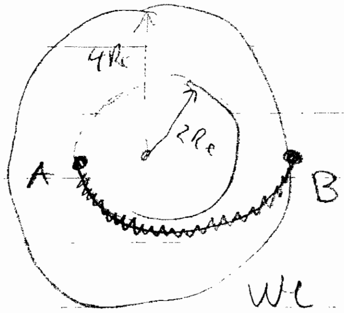
$$\Rightarrow m\ddot{s} = f - mg\sin(s/R)$$

$$\Rightarrow m\ddot{s} + \frac{I\ddot{s}}{b} = -mg\sin(s/R) \Rightarrow \ddot{s} + \frac{\cancel{m}g}{\cancel{m} + \frac{2}{5}\frac{\cancel{m}\cancel{b^2}}{\cancel{b^2}}}\sin\left(\frac{s}{R}\right) = 0$$

or $\ddot{s} + \frac{5}{7}g\sin(s/R) = 0$

For small displacements $\sin\left(\frac{s}{R}\right) \to \frac{s}{R} \Rightarrow \omega_0^2 = \frac{5}{7}\frac{g}{R} \Rightarrow T = 2\pi\left(\frac{7}{5}\frac{R}{g}\right)^{1/2}$

5. Hohmann transfer from circular orbit radius $2R_E$ to circular orbit radius $4R_E$ along an elliptical orbit. Find $\Delta\vec{v}$ at A.



For the circular orbits we use Newton's law to write

$$\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow v = \sqrt{\frac{GM}{R}}$$

(M is the Earth's mass)

So $v_{A,0} = \sqrt{\frac{GM}{2R_E}}$. For the elliptical orbit we consider perigee and apogee (A & B) where $\dot{r} = 0$ and conserve angular momentum and energy:

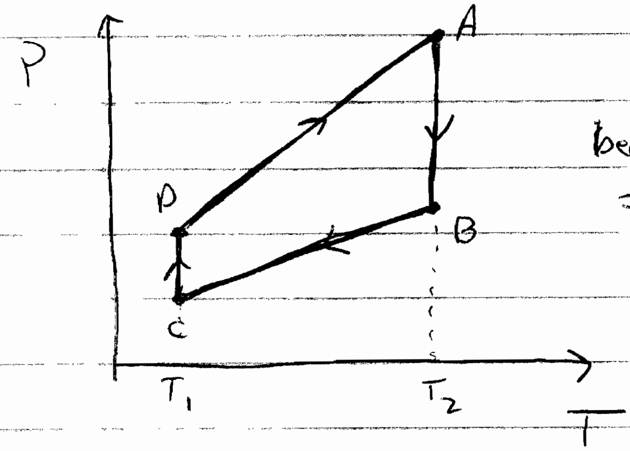
ℓ: $mv_A r_A = mv_B r_B = mv_A 2R_E = mv_B 4R_E \Rightarrow v_B = v_A/2$

E: $$\frac{\ell^2}{2mr_A^2} + \underbrace{\frac{1}{2}m\dot{r}_A^2}_{\text{zero (at apogee and perigee)}} - \frac{GMm}{r_A} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A} = \frac{1}{2}mv_B^2 - \frac{GMm}{r_B}$$

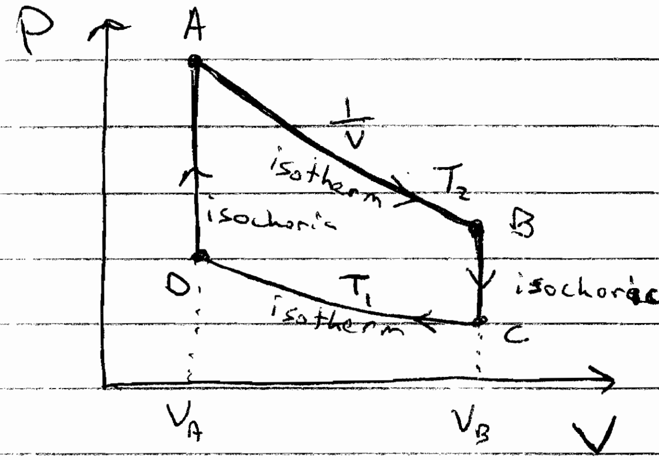
We substitute $v_B = v_A/2$ and $r_A = 2R_E$, $r_B = 4R_E$ and find $v_A = \sqrt{\frac{2}{3}\frac{GM}{R_E}}$. Note that both $\vec{v}_{A,0}$ and $\vec{v}_A$ are tangent to the motion, so $\Delta\vec{v}$ is too, with magnitude

$$\Delta v = v_A - v_{A,0} = \sqrt{\frac{GM}{R_E}}\left(\sqrt{\frac{2}{3}} - \sqrt{\frac{1}{2}}\right)$$

6. Reversible Engine



becomes $\Longrightarrow$



b) $e = \dfrac{W_{net}}{Q_{in}}$

$$W_{net} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= nRT_2 \ln\frac{V_B}{V_A} + 0 + nRT_1 \ln\frac{V_A}{V_B} + 0$$

$$= nR \ln\frac{V_B}{V_A}\left(T_2 - T_1\right)$$

because $\Delta U \propto \Delta T = 0$ because $W = 0$, $Q = \Delta U$

$$Q_{in} = Q_{AB} + Q_{DA} = W_{AB} + \Delta U_{DA}$$

$$= nRT_2 \ln\frac{V_B}{V_A} + \frac{3}{2}nR\left(T_2 - T_1\right)$$

$$\therefore e = \frac{\cancel{nR} \ln\frac{V_B}{V_A}(T_2 - T_1)}{\cancel{nR}T_2 \ln\frac{V_B}{V_A} + \frac{3}{2}\cancel{nR}(T_2 - T_1)} = \boxed{\frac{\ln\frac{V_B}{V_A}}{\ln\frac{V_B}{V_A} + \frac{3}{2}\left(1 - \frac{T_1}{T_2}\right)}\left[1 - \frac{T_1}{T_2}\right.}$$

c) $e_c = 1 - \frac{T_1}{T_2}$; looking at (b) we see that the factor multiplying $\left(1 - \frac{T_1}{T_2}\right)$ is always less than 1. Therefore $e < e_c$.

(7) Easiest to use Lorentz transform

$t_A = 0 \qquad x_A = 480$

$t_B = 5\mu s \qquad x_B = 1200$

$$x_B' - x_A' = 0 = \gamma\left[(x_B - x_A) - v(t_B - t_A)\right]$$

$$\Rightarrow v = \frac{x_B - x_A}{t_B - t_A} = 1.44 \times 10^8 \text{ m/sec} \quad \Rightarrow \gamma = 1.14$$

$$t_B' - t_A' = \gamma\left(5\mu\text{sec} - \frac{(1200-480)v}{c^2}\right)$$

$$= 4.39\,\mu\text{sec}$$

or $t_B - t_A = \gamma\left[t_B' - t_A' + \underbrace{\cancel{(x_B' - x_A')}}_{0}\frac{v}{c^2}\right]$

$$= \gamma\left[t_B' - t_A'\right]$$

$$\Rightarrow t_B' - t_A' = \frac{5}{1.14} = 4.39\,\mu\text{sec}$$