Examination Cover Sheet

Princeton University Undergraduate Honor Committee

January 18, 2008

Course Name & Number: PHY105	Date: January 19, 2008
Professors: Chris Herzog, Lyman Page, Jason Petta	Time: 9:00 AM

This examination is administered under the Princeton University Honor Code. Students should sit one seat apart from each other, if possible, and refrain from talking to other students during the exam. All suspected violations of the Honor Code must be reported to the Honor Committee Chair at honor@princeton.edu.

The checked items are permitted for use on this examination. Any item that is not checked may not be used and should not be in your working space. Assume items not on this list are not allowed for use on this examination. Please place items you will not need out of view in your bag or under your working space at this time. University policy does not allow the use of electronic devices such as cell phones, PDAs, laptops, MP3 players, iPods, etc. during examinations. Students may not wear headphones during an examination.

- ☐ Course textbooks: No
- ☐ Course Notes: No
- ☐ Other books/printed materials: No
- ☒ Formula Sheet: Yes, but only the one provided
- ☐ Comments on use of printed aids: None

Students may only leave the examination room for a very brief period without the explicit permission of the instructor. The exam may not be taken outside of the examination room.

This exam is a timed examination. You will have

3 hours 0 minutes

to complete this exam.

During the examination, the Professor will be outside the door.

On the cover of your first booklet, write and sign the Honor Code pledge: "I pledge my honor that I have not violated the Honor Code during this examination"

This exam consists of **seven** problems. When we begin, check to see that this copy of the exam has all seven. Use the same exam booklet for all problems, continuing to another booklet if necessary. **Print** your name on **each** booklet as you start it.

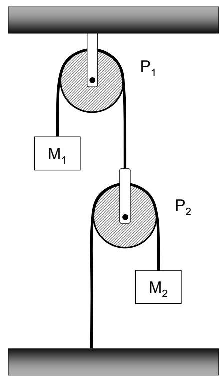
USE OF CALCULATORS IS NOT PERMITTED!!

At the end of the exam, indicate clearly on the cover of your first exam booklet how many booklets you used.

Some useful test-taking hints:

- You may not be able to complete every problem. Keep moving – do what you know first.
- Make your answer clear by circling it.
- Use symbols rather than numbers wherever possible and CHECK UNITS.
- Whenever possible, check whether an answer or intermediate result makes sense before moving on.
- There is a list of formulas on the last page. You can tear it off. Use it as a reminder of details. Don't try to do problems by searching through the sheet!
- If you get stuck on an early part of a problem, check the later parts — some may be independent and doable.
- If you get stuck on an early part of a problem, and a later part depends on it, **clearly** define a symbol for the unknown answer and use it in later parts. Note: this is an act of desperation – we often give multiple parts to guide you through a problem.
- **Show your work!**

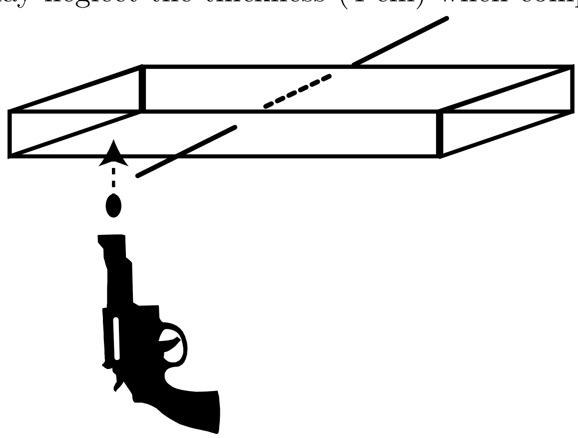
1. Double Atwood System [20 pts]. A double Atwood system consists of masses M_1 and M_2, which are connected by massless strings, as shown below. Assume that the pulleys, P_1 and P_2, are massless and frictionless, and that the string is massless. The wide grey bands indicate the ceiling and floor and are fixed.



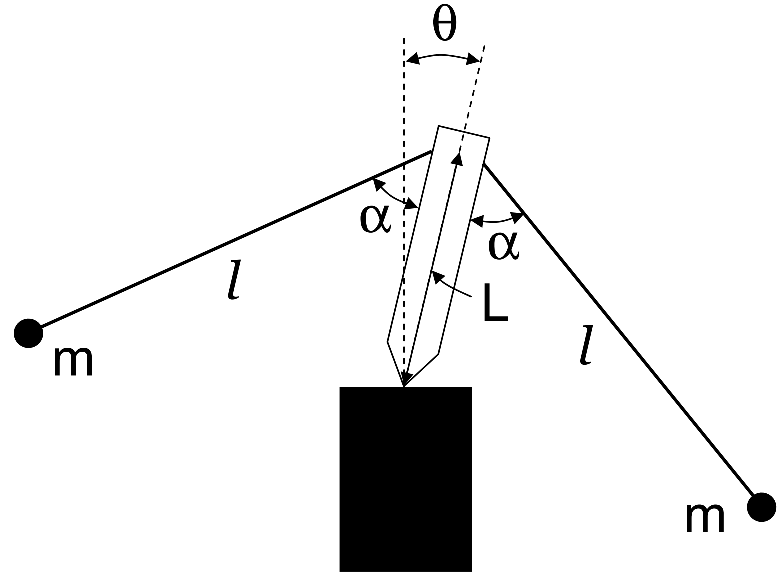
a) [10 pts] Call a_1 the acceleration of M_1 and a_2 the acceleration of M_2. What is the relation between a_1 and a_2? Clearly label your coordinate system and show your work.

b) [10 pts] What is a_1 in terms of M_1, M_2, and g?

2. Firing Block [20 pts]. A block of wood of mass m_w and dimensions 20 cm length by 8 cm width by 2 cm thick has a massless rod through its center of mass and perpendicular to one side. The rod is set atop some supports so that the two large dimensions of the block are in a plane parallel to the ground as shown. A gun is fired from below the block so that the bullet of initial velocity v_b and mass m_b strikes the block roughly 8 cm out from the center along its length but centered in width. The bullet is embedded in the wood. The block rises and spins as a result of the off-center shot. In the limit of $m_b << m_w$, how high does its center of mass rise? You may neglect the thickness (4 cm) when computing the height.



3. Teeter Toy [30 pts]. A teeter toy, shown below, consists of two weights of mass m, which hang from a peg attached to two arms at a fixed angle α. We wish to determine the stability of the teeter toy when the peg is rotated from the vertical by an angle θ. You may consider the arms and peg as massless.



a) [10 pts] What is the potential energy, $U(\theta)$? For simplicity, take the zero of the gravitational potential energy to coincide with the pivot point.(Hints: Use the identity for $\cos(\alpha \pm \beta)$ from the formula sheet to simplify your expression, use only the angles and lengths in the diagram, and notice that the weights must move on a circle.)

b) [5 pts] For what value(s) of θ is the system in equilibrium?

c) [5 pts] What is the condition on L, l, and α for the equilibrium to be stable?

d) [10 pts] In general, the potential energy, U, and the kinetic energy, K, of a harmonic oscillator can be written as:

$$U = \frac{1}{2}Aq^2 + \text{const.} \tag{1}$$

and

$$K = \frac{1}{2}B\dot{q}^2 \tag{2}$$

where A and B are constants. How is the oscillation frequency related to A and B? What is the frequency of small oscillations of the teeter toy?

4. Stepin' Sphere [20 pts]. A sphere of radius R and mass m rolls without slipping toward a vertical step of height h where $h < R$. We want to find the initial rolling velocity, v_0, at which the ball just makes it over the step. In the collision with the step, you may assume that the ball briefly "sticks" to the edge until its center of mass is directly over the edge. The moment of inertia of a sphere is $2mR^2/5$ about its center of mass.

a) [5 pts] What is the angular momentum of the ball with respect to the edge of the step before the collision?

b) [15 pts] What is the initial velocity required so that the ball just climbs over the step?

5. Strange Circles [20 pts]. In polar coordinates (r, θ), a particle of mass m moves in a circle $r = 2R\cos\theta$ $(-\pi/2 \leq \theta \leq \pi/2)$ under the influence of a central force potential $U(r) = -K/r^{\alpha}$. Notice that this orbit passes through the origin. Though the orbit is unstable, this fact does not concern us. There are no relativistic effects.

a) [10 pts] What is the total energy, E, of the orbit?

b) [5 pts] What must α be?

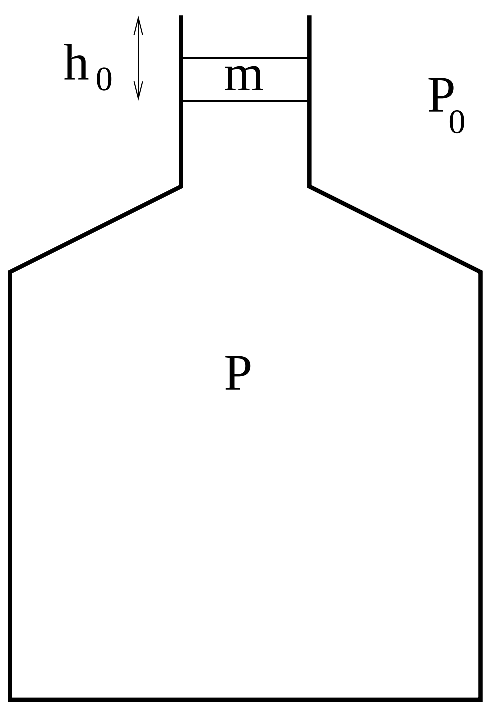
c) [5 pts] What is its angular momentum, l, in terms of m, R, & K?

6. Bottle Tunes [20 pts]. A bottle of total volume V has a narrow cylindrical neck at the top of cross sectional area A. The bottle sits at equilibrium in a room at a pressure P_0 and fixed temperature. The bottle is sealed by placing a light cylinder of mass m in the neck. The cylinder can slide frictionlessly up and down inside the neck but does not let gas pass from one side to the other.

a) [10 pts] After a suitable time, a new equilibrium is reached with the cylinder inside the neck but a distance h_0 from the top and the gas in the bottle at the same temperature as the room. What is h_0 in terms of V, A, P_0, m, and g? You may assume that $mg << AP_0$.

b) [10 pts] If the cylinder is displaced a small distance away from equilibrium, x, it oscillates with frequency ω. Assuming the compression of the gas in the bottle is adiabatic (unlike the isothermal compression on PS10), what is ω? You may assume that $Ax << V$.

[No points] This oscillation is meant to be a toy model of the effect of blowing on a soda bottle where the cylinder of mass m is the air mass inside the neck of the bottle. Given $A = \pi/4$ cm^2, $V = 5.6$L, $m = 10^{-3}$ g, and a pressure of about one atmosphere, $P_0 = 10^6$ dyn cm^{-2}. In the solutions we will post the frequency of oscillation.



7. The Pole in the Barn [20 pts]. A farmer owns a barn with electric doors on opposite ends that she can close extremely rapidly and simultaneously. The length of the barn is $4l_0/5$ as shown in the figure. A pole vaulter has a pole that has a length l_0 as measured at rest by the farmer.

a) [5 pts] The farmer says to the pole vaulter "if you run fast enough, your pole will contract and just as you pass through the barn I'll be able to close both doors (and then quickly open them) and have you inside for an instant." How fast does the pole vaulter have to run to fit inside the barn and have the doors close?

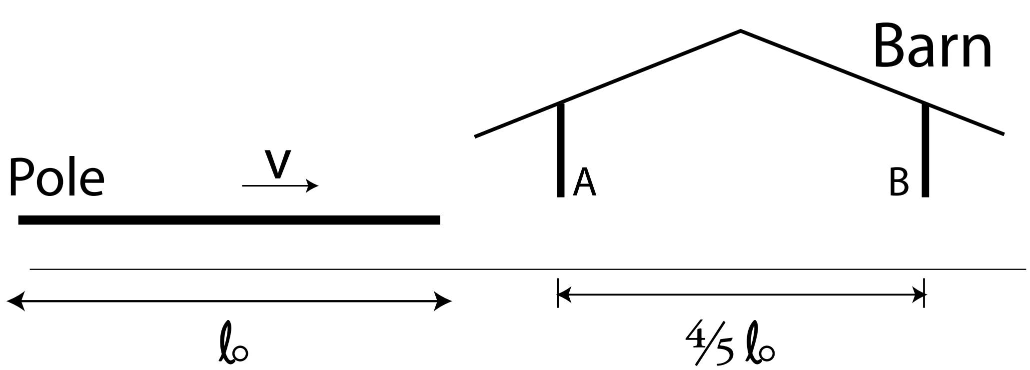
The pole vaulter says "no way! according to me the barn will be Lorentz contracted and I'll smash the doors when you close them for that instant." Assume that the farmer and pole vaulter sync up their observations when the back of the pole is aligned with the left hand ("A") door.

Call the pole vaulter coordinates x' and t' and express all times and distances in terms of l_0 and c. For the pole vaulter:

b) [5 pts] When does the "B" (right hand) door close and then quickly open?

c) [5 pts] How long does the pole vaulter think he is in the barn?

d) [5 pts] Is the pole vaulter correct? Does any wood get splintered? Briefly describe the resolution to this apparent paradox.



Physics 105 Formula Sheet

$$\mu = \frac{Mm}{M+m}, \qquad U_{\text{eff}} = \frac{l^2}{2\mu r^2} + U(r), \qquad \mathbf{F} = -\frac{GMm}{r^2}\,\hat{\mathbf{r}}, \qquad U = -\frac{GMm}{r}, \qquad T^2 = \frac{\pi^2 \mu A^3}{2C}$$

$$a_c = \frac{v^2}{r}, \qquad r = \frac{r_0}{1 - \epsilon\cos\theta}, \qquad r_0 \equiv l^2/\mu C, \qquad \epsilon \equiv \sqrt{1 + \frac{2El^2}{\mu C^2}}, \qquad C \equiv GMm$$

$$A = -C/E$$

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0 \quad \left(\text{or} = \frac{F}{m}\right)$$

$$\ddot{x} + \omega_0^2 x = 0, \qquad \omega_0^2 = k/m, \qquad x = A\cos(\omega_0 t + \phi), \qquad 2\pi f = \omega, \qquad T = 1/f$$

$$e^{i\phi} = \cos\phi + i\sin\phi, \qquad z = x + iy = re^{i\phi}, \qquad r^2 = zz^*, \qquad \tan\phi = \frac{\text{Im}z}{\text{Re}z}$$

$$y(x,t) = f(x \mp vt), \qquad y(x,t) = A\cos(kx \mp \omega t) \qquad v = \frac{\lambda}{T} = \frac{\omega}{k}, \qquad \lambda = \frac{2\pi}{k}$$

$$f'_{sound} = f_0(v_s \pm v_{rec})/(v_s \mp v_{src})$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, \qquad \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta, \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\text{Lorentz transform}: \; x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t' = \gamma(t - xv/c^2)$$
$$x = \gamma(x' + vt'), \quad y = y', \quad z = z', \quad t = \gamma(t' + x'v/c^2)$$

$$\beta = v/c, \qquad \gamma = 1/\sqrt{1-\beta^2}, \qquad u_x = (u'_x + v)/(1 + vu'_x/c^2), \qquad u_y = u'_y/\gamma(1 + vu'_x/c^2)$$

$$E = \gamma m_0 c^2, \qquad p = \gamma m_0 v, \qquad E^2 = p^2c^2 + m_0^2c^4, \qquad E = pc \; (for\; m_0 = 0)$$

For a planar object rotationally symmetric about z:

$$\overleftrightarrow{\text{I}} = \begin{pmatrix} \int \rho y^2\, dV & 0 & 0 \\ 0 & \int \rho x^2\, dV & 0 \\ 0 & 0 & \int \rho(x^2+y^2)\, dV \end{pmatrix}$$