PHYSICS 301 MIDTERM EXAM
October 22, 2008

This exam consists of **three** problems on one page. Use the same exam booklet for all problems, continuing to another booklet if necessary. **Print** your name on **each** booklet as you start it. On the cover of your first booklet, **COPY** and **SIGN** the following pledge:

I pledge my honor that I have not violated the Honor Code during this examination.

At the end of the exam, indicate clearly on the cover of your first exam booklet how many booklets you used.

Some useful test-taking hints:

- You may not be able to complete every problem. Keep moving – do what you know first.
- Make your answer clear by circling it.
- Use symbols rather than numbers wherever possible and check units.
- Whenever possible, check whether an answer or intermediate result makes sense before moving on.
- If you get stuck on an early part of a problem, check the later parts — some may be independent and doable.
- If you get stuck on an early part of a problem, and a later part depends on it, **clearly** define a symbol for the unknown answer and use it in later parts. However, keep in mind that we often give multiple parts to guide you through a problem.

To get full credit you need to show your work!

NO CALCULATORS!

The exam will last 2 hours: 7:30-9:30 PM

Good luck!

Problem 1. [50 points] A crude model of a rubber band (a 1D elastic polymer) consists of a chain of N segments each of length a and mass m. Imagine that the rubber band is under some tension W and that left going segments add energy Wa and right going segments add energy $-Wa$. At temperature τ the rubber band has length $\ell = -U/W$, where U is the net energy.

a) What is the partition function for the rubber band assuming each segment is distinguishable?

b) What is the energy of the rubber band in terms of τ, N, W, and a.

c) What is the Helmholtz free energy?

d) What is the entropy?

e) In the limit that $Wa << \tau$ (or $\ell << Na$), find an expression for the spring constant in terms of τ, N, and a.

Problem 2. [40 points] Our model of an Einstein Solid consists of 3N independent quantized harmonic oscillators, with 3 orthogonal oscillators per atom.

a) What is the partition function per oscillator (ignoring the zero point energy)?

b) What is the dimensionless entropy per oscillator? It may help to start with the Helmholtz free energy.

c) What is the conventional (with units of J/K) C_V for the solid?

d) In the high temperature limit, what is C_V and does it agree with expectations? Please explain.

Problem 3. [40 points] We have seen that in general the entropy is proportional to the number of particles. For example, for photons $\sigma/V \approx 3.6N/V$ where V is the volume. We may think of a black hole as a region in space with the maximum possible number of particles consistent with its mass (or energy). A black hole has radius $r_S = 2GM/c^2$ where M is the black hole mass, G is Newton's constant, and the "S" subscript is for Schwartzchild. The black hole energy is $U = Mc^2$ and is directly related to r_S.

a) To find the maximum number of particles, we want to consider how many of the least massive (or energetic) particles the black hole can accommodate. Approximating the black hole as a cube of side $2r_S$, how many photons of wavelength $4r_S$ would "fit in" the black hole? Call your answer σ_{BH}/π^2, the entropy of the black hole divided by π^2.

b) What is the temperature of a black hole? Please give your answer in terms of G, M, c, k, & h.

c) Assuming an emissivity of unity, how long will it take a black hole of initial mass M_0 to radiate away all of its energy and disappear? Please leave your answer in an analytic form. You should be able to express your answer in terms of G, h, & c.

d) If the Large Hadron Collider at CERN produced black holes they might have an energy of ~10 TeV (10^{13} eV), near the center of mass energy of the collider. How long would such a black hole live in seconds? A rough estimate, within a factor of 10^4, will suffice. Numerical values of constants are given on the formula page.

Physics 301 Formula Sheet

$$dF = \left.\frac{\partial F}{\partial \tau}\right|_{V,N} d\tau + \left.\frac{\partial F}{\partial V}\right|_{\tau,N} dV + \left.\frac{\partial F}{\partial N}\right|_{\tau,V} dN = -\sigma\; d\tau - pdV + \mu dN \tag{1}$$

$$dU = \left.\frac{\partial U}{\partial \sigma}\right|_{V,N} d\sigma + \left.\frac{\partial U}{\partial V}\right|_{\sigma,N} dV + \left.\frac{\partial U}{\partial N}\right|_{\sigma,V} dN = \tau d\sigma - pdV + \mu dN \tag{2}$$

$$d\sigma = \left.\frac{\partial \sigma}{\partial U}\right|_{V,N} dU + \left.\frac{\partial \sigma}{\partial V}\right|_{U,N} dV + \left.\frac{\partial \sigma}{\partial N}\right|_{U,V} dN = dU/\tau + pdV/\tau - \mu dN/\tau \tag{3}$$

$$F = U - \tau\sigma = -\tau \ln Z \tag{4}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \tanh(x) = \frac{\sinh(x)}{\cosh(x)} \tag{5}$$

$$\text{Stirling's formula} : N! \approx N^N e^{-N}\sqrt{2\pi N} \qquad \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad (\text{for } x < 1) \tag{6}$$

$\sigma_{SB} = \frac{\pi^2 k^4}{60\hbar^3 c^2} = 5.67 \times 10^{-8}$ W/m^{2}K^4
$c^5/G^2 = 3.63 \times 10^{52}$ J/s $= 2.27 \times 10^{71}$ eV/s
$\hbar = 1.05 \times 10^{-34}$ J s $= 6.6 \times 10^{-16}$ eV s
$k = 1.38 \times 10^{-23}$ J/K $= 8.63 \times 10^{-5}$ eV/K
$c = 3 \times 10^{8}$ m/s
$G = 6.67 \times 10^{-11}$ m^3/s^2 kg
Bohr magneton $= 9.27 \times 10^{-24}$ J/T $= 5.79 \times 10^{-5}$ eV/T
Proton mass $= 1.67 \times 10^{-27}$ kg or 939 MeV Electron mass $= 9.11 \times 10^{-31}$ kg or 511 keV
Solar mass $= 1.99 \times 10^{33}$ kg Solar radius $= 6.96 \times 10^{8}$ m
Earth mass $= 5.98 \times 10^{24}$ kg Earth radius $= 6.37 \times 10^{6}$ m
Earth Sun distance $= 1.5 \times 10^{11}$ m

It may be useful to know that 300 K corresponds to 1/40 eV.