# PHYSICS 301 MIDTERM EXAM

October 29, 2009

This exam consists of **four** problems on two pages. Use the same exam booklet for all problems, continuing to another booklet if necessary. **Print** your name on **each** booklet as you start it. On the cover of your first booklet, **COPY** and **SIGN** the following pledge:

*I pledge my honor that I have not violated the Honor Code during this examination.*

At the end of the exam, indicate clearly on the cover of your first exam booklet how many booklets you used.

Some useful test-taking hints:

- You may not be able to complete every problem. Keep moving – do what you know first.
- Make your answer clear by circling it.
- Use symbols rather than numbers wherever possible and check units.
- Whenever possible, check whether an answer or intermediate result makes sense before moving on.
- If you get stuck on an early part of a problem, check the later parts — some may be independent and doable.
- If you get stuck on an early part of a problem, and a later part depends on it, **clearly** define a symbol for the unknown answer and use it in later parts. However, keep in mind that we often give multiple parts to guide you through a problem.

**To get full credit you need to show your work!**

**NO CALCULATORS!**

**The exam will last 2 hours: 7:30-9:30 PM**

**Good luck!**

**Problem 1.** [10 points] Our DNA is comprised of a linear sequence of base pairs one element of which is A, C, G, or T (adenine, cytosine, guanine, and thymine). For this problem you may ignore the fact that the bases are paired up and you may think of just one side of the double stranded structure. Thus, think of a linear sequence comprised of A, C, G, and T.

a) If we have $n$ molecules of each at our disposal how many different kinds of DNA can be built using $4n$ bases total?

b) A DNA polymer can have $4n = 10^6$ base pairs. Give an approximate numerical expression for the number of different possible kinds of DNA. Express your answer in the form of $2^y$ where $y$ is the number you are seeking.

**Problem 2.** [30 points] When electromagnetic radiation is confined to a cavity with reflecting walls, it can oscillate only at discrete frequencies determined by the boundary conditions. The spatial pattern of the oscillation is called a "mode." At each allowed frequency, $\omega$, in other words in each mode, there may be $n$ photons when $n$ can assume any value. The possible energy states of the mode are thus given by $\epsilon = n\hbar\omega$, where we have ignored the zero point energy.

a) What is the probability that the radiation at temperature T has $n$ quanta in the mode oscillating at $\omega$?

b) What is the average occupancy of this mode at temperature T?

c) What is the relative rms fluctuation of the number of photons, $(\overline{(\Delta n)^2}/\overline{n}^2)^{1/2}$, in this mode at temperature T and large mode occupation number $n$? Please express your answer as a percentage.

**Problem 3.** [30 points] The equation of state of a gas can be written in the form

$$p = nkT(1 + nB_2) \tag{1}$$

where $p$ is the pressure, $n = N/V$, $T$ is the temperature, and $B_2 = B_2(T)$ is the second virial coefficient. $B_2$ is an increasing function of temperature and we may take $nB_2 << 1$.

a) What is $\partial E/\partial V|_T$ in terms of the quantities in the equation of state and their derivatives?

b) Recall that the Joule-Thomson coefficient is given by

$$\mu_{JT} \equiv \left.\frac{\partial T}{\partial p}\right|_H . \tag{2}$$

What is $\mu_{JT}$ in terms of $C_p$, $N$, $T$, $B_2$ and $\partial B_2/\partial T$? Under what conditions will the gas cool upon expansion? (Hint: A useful approximation is $V = N(kT/p + B_2)$.)

**Problem 4.** [30 points] In class you found that when stretched the temperature of a rubber band increases. We investigate this finding here. The stretch may be considered adiabatic and isochoric (constant volume) even though the length changes. In other words, $\partial \mathrm{T}/\partial \ell|_{\mathrm{S,V}} > 0$, where $\ell$ is the length. In the high temperature limit, we found that the entropy is given by

$$S = S_0 - \frac{k\ell^2}{Na^2} \tag{3}$$

where $a$ is the length of each molecule, $k$ is Boltzmann's constant, and $N$ is the number of molecules. In class, we consider only the entropy associated with the orientation of the molecules and not that associated with the molecules themselves. We may crudly include the intrinsic entropy of the molecules by letting $S_0$ be a monotonically increasing function of temperature alone. Thus we let $S_0 = S_0(T)$. The related fundamental identity is:

$$TdS = dE - f\,d\ell \tag{4}$$

where $f$ is stress in the rubber band (analogous to the negative of the pressure).

a) Using the information given above, derive the equation of state of the rubber band in terms of $f$.

b) Find an expression for $\partial \mathrm{T}/\partial \ell|_{\mathrm{S,V}}$ in terms of $\partial \mathrm{S}/\partial \mathrm{T}|_{\ell,\mathrm{V}}$, $k$, $N$, $a^2$, and $\ell$.

c) Find an expression for $\partial \mathrm{T}/\partial \ell|_{\mathrm{S,V}}$ in terms of the more easily measured $\partial \mathrm{f}/\partial \mathrm{T}|_{\ell,\mathrm{V}}$, heat capacity at constant $\ell$, and $T$.

N.B. A number of typos/clarifications pointed out and discussed during the exam have been fixed here.

## Physics 301 Formula Sheet

$$dE = \left.\frac{\partial E}{\partial S}\right|_{V,N} dS + \left.\frac{\partial E}{\partial V}\right|_{S,N} dV + \left.\frac{\partial E}{\partial N}\right|_{S,V} dN = TdS - pdV + \mu dN \tag{5}$$

$$F = E - TS = -kT \ln Z \tag{6}$$

$$dF = \left.\frac{\partial F}{\partial T}\right|_{V,N} dT + \left.\frac{\partial F}{\partial V}\right|_{T,N} dV + \left.\frac{\partial F}{\partial N}\right|_{T,V} dN = -S\ dT - pdV + \mu dN \tag{7}$$

$$H = E + pV \tag{8}$$

$$dH = \left.\frac{\partial H}{\partial S}\right|_{p,N} dS + \left.\frac{\partial H}{\partial p}\right|_{S,N} dp + \left.\frac{\partial H}{\partial N}\right|_{S,p} dN = T\ dS + V dp + \mu dN \tag{9}$$

$$G = E + pV - TS \tag{10}$$

$$dG = \left.\frac{\partial G}{\partial T}\right|_{p,N} dT + \left.\frac{\partial G}{\partial p}\right|_{T,N} dp + \left.\frac{\partial G}{\partial N}\right|_{T,p} dN = -S\ dT + V dp + \mu dN \tag{11}$$

$$\left.\frac{\partial T}{\partial V}\right|_S = -\left.\frac{\partial p}{\partial S}\right|_V \qquad \left.\frac{\partial T}{\partial p}\right|_S = \left.\frac{\partial V}{\partial S}\right|_p \qquad \left.\frac{\partial S}{\partial V}\right|_T = \left.\frac{\partial p}{\partial T}\right|_V \qquad \left.\frac{\partial S}{\partial p}\right|_T = -\left.\frac{\partial V}{\partial T}\right|_p \tag{12}$$

$$V \quad F \quad T \tag{13}$$

$$E \qquad\quad G \tag{14}$$

$$-S \quad H \quad -P \tag{15}$$

$$\tag{16}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \tanh(x) = \frac{\sinh(x)}{\cosh(x)} \tag{17}$$

$$\text{Stirling's formula} N! \approx N^N e^{-N}\sqrt{2\pi N} \qquad \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad (\text{for } x < 1) \tag{18}$$

$\sigma_{SB} = \frac{\pi^2 k^4}{60\hbar^3 c^2} = 5.67 \times 10^{-8}$ W/m$^2$K$^4$
$\hbar = 1.05 \times 10^{-34}$ J s $= 6.6 \times 10^{-16}$ eV s
$k = 1.38 \times 10^{-23}$ J/K $= 8.63 \times 10^{-5}$ eV/K
$c = 3 \times 10^8$ m/s
$G = 6.67 \times 10^{-11}$ m$^3$/s$^2$ kg
Bohr magneton $= 9.27 \times 10^{-24}$ J/T $= 5.79 \times 10^{-5}$ eV/T
Proton mass $= 1.67 \times 10^{-27}$ kg or 939 MeV  Electron mass $= 9.11 \times 10^{-31}$ kg or 511 keV
Solar mass $= 1.99 \times 10^{33}$ kg  Solar radius $= 6.96 \times 10^8$ m
Earth mass $= 5.98 \times 10^{24}$ kg  Earth radius $= 6.37 \times 10^6$ m
Earth Sun distance $= 1.5 \times 10^{11}$ m

It may be useful to know that 300 K corresponds to 1/40 eV.