Physics 304 Final Exam May 22, 2004

180 minutes

The exam is "open book" but **not** "open notes." You will not need a calculator.

Write your answers in an exam book. Please be sure to put your name on the front of the book! Show your work, explain your reasoning, and please box your final answers.

There are 4 problems with point values indicated. The total point value is 100.

Good luck.

After you have completed the exam, write and sign the honor pledge on the front of the exam book:
"I pledge my honor that I have not violated the Honor Code during this examination."

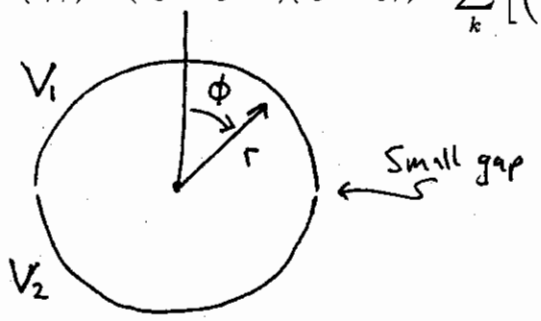
Possibly useful integrals:

$$\int_0^{\pi} \sin^3\theta\, d\theta = \frac{4}{3} \qquad \int \cos^2 bx\, dx = \frac{\sin 2bx}{4b} + \frac{x}{2} \qquad \int \sin bx\, dx = -\frac{\cos bx}{b}$$

$$\int \cos bx\, dx = \frac{\sin bx}{b} \qquad \int_0^{\pi/2} \sin x \cos^2 x\, dx = \frac{1}{3}$$

Problem 1 [20 pts]. A long hollow cylindrical conductor of radius a is divided in two parts by a plane through the axis. The two parts are separated by a small ignorable gap and held at potentials V_1 and V_2 as shown in the figure below. What is the potential $V(r, \phi)$ at any point within the cylinder? You may leave your answer as an infinite series. You may recall that by solving Laplace's equations in cylindrical coordinates using separation of variables, one finds:

$$V(r,\phi) = (a_0 + b_0 \ln r)(c_0 + d_0\phi) + \sum_k \left[\left(a_k r^k + \frac{b_k}{r^k}\right)\cos k\phi + \left(c_k r^k + \frac{d_k}{r^k}\right)\sin k\phi\right].$$



Problem 2 [30 pts]. A spherical shell has mass m, radius a, and a total charge Q uniformly distributed over its surface. The shell is spun up to angular velocity $\omega = \omega_0 \hat{z}$ over a period of time $\tau >> a/c$, where c is the speed of light; thus radiation may be ignored.

a)What are the E and B fields throughout space? Express any changing field as a function of $d\omega/dt$ and be sure to include directions. Feel free to use the results from example 5.11 (pg 237).

b)What is the total electromagnetic angular momentum?

c)Find the flow of angular momentum from the surface of the sphere at $r = a$ through a spherical surface just outside of $r = a$ using the stress energy tensor, $\mathbf{T}$. From the discussion on pg 356, we infer that the flow is given by $-\int (\mathbf{r} \times \mathbf{T}) \cdot d\mathbf{a}$. It may help you to know that $(\mathbf{r} \times \mathbf{T}_E) \cdot \hat{r} = \epsilon_0 (\mathbf{r} \times \mathbf{E}) E_r$ where $\mathbf{T}_E$ is the electric field stress energy tensor.

d)What is the relation between parts (b) and (c)?

Problem 3 [20 pts]. A current I carried by freely moving electrons runs through a cylindrical wire with radius a. Assume that the electrons are moving with a uniform velocity v and that the total charge per unit length in the wire vanishes (in the lab frame). This is similar to Griffith 5.38 which you did for homework. We are asking you to extend your solution.

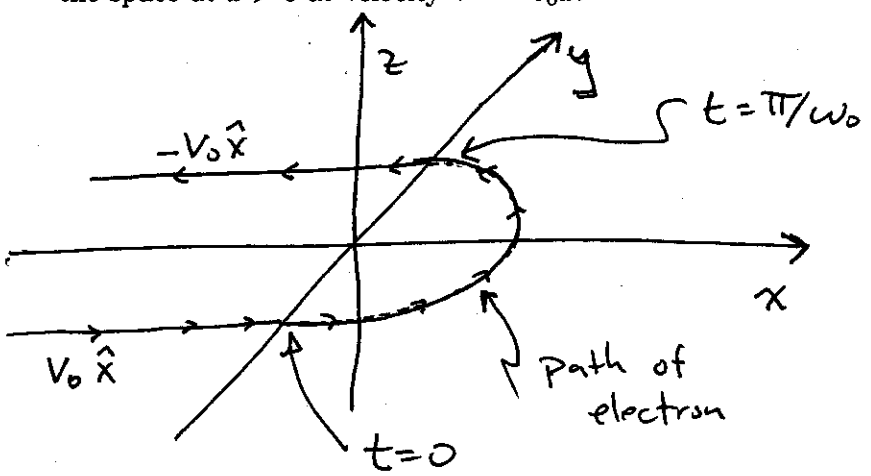
a)What is the radial profile of the electric field? Make a sketch and label the essential features.

b)What is the voltage between the center and edge of the cylinder?

Problem 4 [30 pts]. A magnetic field $\mathbf{B}(x)$ is given by:

$$\mathbf{B}(x) = 0 \quad \text{for } x < 0 \tag{1}$$
$$\mathbf{B}(x) = B_0\hat{z} \text{ for } x > 0 \tag{2}$$

There is no electrostatic field. A non-relativistic electron of mass m_e initially moves at constant velocity $\mathbf{v} = v_0\hat{x}$ in the space $x < 0$. As shown in the figure, at $t = 0$ the electron enters the space $x > 0$, gyrates in a semi-circle with angular frequency $\omega_0 = eB/m_e$ for a time π/ω_0, and then exits the space at $x > 0$ at velocity $\mathbf{v} = -v_0\hat{x}$.



a)What $dP(t)/d\Omega$ does an observer on the x-axis at $\mathbf{x} = r\hat{x} >> c/\omega_0$ observe? Sketch $dP(t)/d\Omega$ as a function of time and quantitatively label key features.

b)Another observer sits on the z axis at $\mathbf{z} = r\hat{z} >> c/\omega_0$. Again, sketch $dP(t)/d\Omega$ as a function of time and quantitatively label key features.

c)What is the total energy emitted by the electron?